Effect of Sampling Rates Variation during Cylindricity Error Evaluation

Amira A. Khattab, Yasmine Abouelseoud, Mohammad A. Younes

Abstract— **Cylindrical parts are critical elements in many engineering equipment. To perform correctly, their dimensions and cylindricity should be within specified tolerances. Cylindrical parts can be checked using several dedicated and general purpose measuring systems. Coordinate Measuring Machines (CMMs) can been used to probe and scan cylindrical surfaces to evaluate dimensions and form errors. For accurate evaluation of cylindricity error using a CMM, several parameters should be taken into consideration such as measurement strategy, sampling rate, and cylindricity evaluation technique.**

The large measurement points required need an efficient evaluation algorithm, and due to their advantages, the Minimum Zone Cylindricity evaluation technique (MZC) and Particle Swarm Optimization technique (PSO) are used in this work. The proposed algorithm was developed using Matlab software and applied for the evaluation of cylindricity error of test cylinders. This paper investigates the effect of the change in sampling rate on the value of estimated cylindricity error when using two different strategies, namely, circles and helical. A comparison between both strategies is presented. The effect of the number of scanned circles and helix turns on the estimated cylindricity error is also investigated. The results showed that sampling rate has more significant effect on the value of estimated cylindricity error than the number of measurement circles or helix turns even for the same number of data points.

Index Terms— **Cylindricity, CMM, Sampling rate, Sampling strategy, optimization**

I. INTRODUCTION

Cylindrical forms are one of the most fundamental features in mechanical designs such as shafts and axles. Cylindrical surfaces can be produced using different production processes. Different factors in the production process may cause cylindrical features to deviate from its nominal shape. Deviations from cylindrical shape will affect the function of the product directly. It is not enough just to measure their diameters or positions, but it is crucial to measure their out of cylindricity as well. So, it is important to design appropriate procedures for assessing cylindricity error and associated uncertainty.

Gene R. Cogorno [1] defines cylindricity as a condition where all points on the surface of a cylinder are equidistant from the axis. The surface being controlled must lie between two coaxial cylinders in which the radial distance between them is equal to the tolerance specified in the feature control frame. Cylindricity is a composite form tolerance that simultaneously controls circularity, straightness, and taper of cylindrical

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features. According to ISO 1101 [2], as well as research works [3, 4, 5], deviations from cylindrical form could be classified as follows: (a) deviation of the centreline, (b) radial deviation, (c) deviation of roundness profiles, and (d) compound deviations. Some of the sources of these deviations are: vibration in the machine tool, tool wear, transmitted errors of machine tool motions, distortion resulting from thermal, pressure or stress effects.

Deviation from cylindricity can be determined by applying different measurement strategies such as: radial section method, generatrix method, helical line method, bird-cage method [5, 6]. Measurement strategies will be discussed in the next sections. Generally cylindricity error can be measured mechanically by different methods such as a vee block and dial indicator, two centres and a dial indicator, and dedicated machines such as roundness testers or coordinate measuring machines.

The work presented by Souza C. C. et al [3] described a procedures to measure the circularity and cylindricity deviations using three different measurement systems, namely; a dial gauge attached to a tailstock device, a manual CMM, and a roundness and cylindricity measurement equipment manufactured by Taylor Hobson, model Talyrond 131. It was concluded that; although the procedures carried out in circularity and cylindricity measurements using different measurement systems are similar, the mathematical models associated with them are different, because each measurement system presents different characteristics and working principles. Different reference cylinders may be used to determine cylindricity error [7, 8], Minimum Zone reference cylinder (MZCY), Least Squares reference cylinder (LSCY), Minimum Circumscribed reference cylinder (MCCY), and Maximum Inscribed reference cylinder (MICY).

Wentao S., Dan Z. [9] used MZC technique for roundness error evaluation and confirmed that among the four methods; only the MZC complies with ISO standards resulting in the minimum roundness error value. The LSC method is robust but does not guarantee the minimum zone solution specified in the standards. Gadelmawla [10] introduced efficient algorithms to evaluate the roundness error using (MCC), (MIC) and (MZC). The results revealed also that roundness error evaluated by (MCC) and (MIC) techniques are larger than those evaluated by (MZC) technique but the evaluation time is larger than that for the (MZC) method.

Since, evaluating cylindricity error involves searching for the two coaxial cylinders containing all measured points with minimum radial separation, it is considered as a minimization problem. Solving for minimum cylindricity error can be transformed to find a group of parameters corresponding to the minimum of the objective function. Such optimization problem can be solved using different optimization techniques such as; Particle Swarm Optimization (PSO) and

Genetic Algorithm (GA). Many researchers used (PSO) to solve different optimization problems. PSO is a population based stochastic optimization technique, developed by Kennedy and Eberhart in 1995 [11, 12].

Compared with genetic algorithm (GA) and immune algorithm (IM), PSO is easier to implement and there are fewer parameters to adjust [13]. Mao J., et al [14], used (PSO) algorithm for uncertainty evaluation of cylindricity errors. One of the advantages found is the faster convergence speed and calculating precision of the PSO algorithm. Meanwhile, the evaluation results are more accurate and in accord with the requirements of the new generation of GPS standard. Zhang *et al* [15] introduced a hybrid (PSO) differential evolution algorithm to calculate the minimum zone cylindricity and conicity errors claiming more accurate and stable results. Cui et al. [16] used PSO to obtain roundness errors by MZC technique, and the results showed also that PSO has a faster convergence than GA does.

Extraction strategies include; Bird-cage extraction strategy, Roundness profile extraction strategy, Generatrix extraction strategy, Points extraction strategy, and helical strategy (Figure 1). When extraction is made by any of the above strategies, only a limited number of sample points of the cylinder are considered [15, 17]. For this reason and because of different instrument designs and specific implementation of the strategies, differences may occur in the measurement results unless care is taken to select a set of points which, for the purpose of the specific assessment, is adequate to represent the cylindrical feature. Barini E. M., et al [18] considered uncertainty analysis during the measurement of complex surfaces using point-by-point sampling on a tactile CMM. The authors studied the effect of four important factors on the uncertainty; sampling density, measurement strategy, probe configuration, and alignment. The study proved that the main factors affecting the results are measurement strategy, sampling density and the interaction of both factors. Vrba, I., et al [19] investigated the manner in which the sampling strategy and the evaluation method influence the assessment of cylindricity error on a coordinate measuring machine. The two parameters considered are sampling strategy and cylindricity evaluation methods. It was concluded that sampling strategy has much greater influence on estimated cylindricity error than the evaluation method.
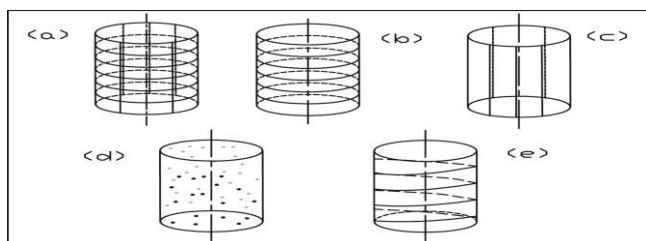

Figure 1 Cylindricity extraction strategies: a- bird cage, b- roundness profile, c- generatix, d- points, and e-helical

Sampling issues that can affect validity of results deal mainly with the location and number of the sampling points. Accurate selection is important when preparing a measurement program for a computer-controlled CMM. Considerations such as: number of sample points, location of sample points, path planning, and probe qualification are very important [20]. The optimum number of sampling points for cylindricity measurement is still somewhat of a question. However,

according to statistical rules, if too few points are taken, the measurement uncertainty will increase. On the other hand, if too many points are measured, the cost of measurement will increase [21].

The traditional method of measuring cylindricity with a CMM is to measure a set of points evenly spaced around the circumference at two or three axial sections [22]. In industry, it is common to keep the sample size as small as possible such as 4-8 points for cylindrical features, whereas most CMMs use six measurement points to determine diameter and cylindricity [3]. However, when using this number, there is a risk of accepting an out-of-specification part [22].

Weckenmann et al [23] stated that, for a comprehensive analysis of the maximum inscribed circle and least square circles, 10- 20 points are required. Ollison T. E. *et al* [20] and Jiang & Chiu [21] also suggested that when the number of measurement points on a cylinder is greater than eight, a confidence interval of 95% is achieved.

Moreover, determination of the number of sections that should be measured on a cylinder is very important for the measurement of cylindricity. Based on available literature the common number of sections used ranges between 2 and 4 sections. Jian & Chiu [21] used three sections while Summerhays [22] used four measurement sections. Recently, computer controlled CMMs are able to scan the cylinder surface and hence a large number of points can be obtained.

This study applies the MZC-PSO for cylindricity error evaluation and investigates the effect of changing the sampling rate on the estimated cylindricity error. Two different measurement strategies are considered; helical strategy and circles strategy. Experiments were carried out using 10 helix turns and 10 circles along the test cylinder axis. The same measurement procedure was repeated three times on each test sample. Three test cylinders were used to evaluate cylindricity errors and measurement procedure was repeated three times on each test sample. Cylindricity error of each cylinder was evaluated at different sampling rates. Furthermore, the effect of the selected number of measurement circles on evaluated error is also investigated.

II. Experimental Work

Measurements were carried on a Zeiss Duramax CNC CMM. The machine maximum permissible error is $\pm (2.4 + (L/300))$ μm at room temperature range of 18 to 22°C, where L is the measuring length in mm. The scanning probe tip is 3 mm in diameter and the probing error is ≤ 0.8 μm as an actual value while the nominal value is (2.4 μm). The measuring software used is Calypso. CMM displacement scale value is 0.0001mm. Figure 2 shows the measurement setup.

Figure. 2 Measurement Setup

III. MATHEMATICAL MODEL OF THE MINIMUM ZONE CYLINDRICITY ERROR

According to the ISO/1101 [2] and Wen X. et al [24], the MZC cylindricity error is expressed as the minimum normal distance between a pair of coaxial cylinders having minimum radial separation and enclosing all the points in the dataset. Figure 3 shows a cylinder illustrated in a three dimensional coordinate system (xyz) [24], where

$$p_i = (x_i, y_i, z_i), \ i = (1, 2, \ldots, N)$$ are the measured points
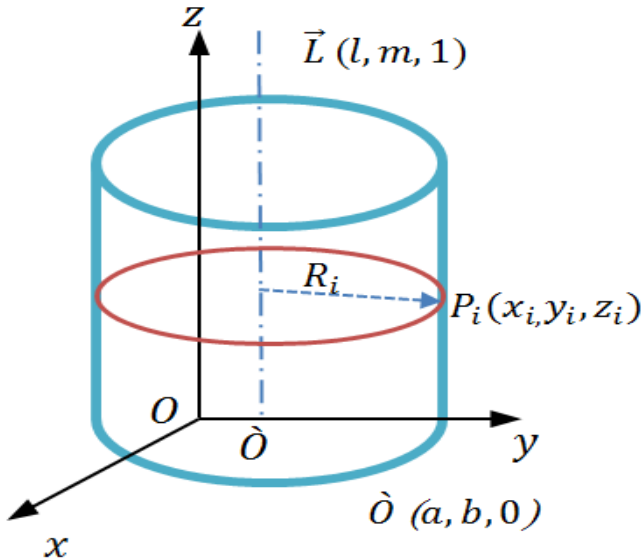
on the cylinder surface.

Figure 3 A cylinder in measurement coordinate system

The cylinder axis L and the coordinate axis z are parallel. $\grave{O}$ $(a, b, 0)$ is the point of intersection between the axis L and the coordinate plane xoy. The axis of cylinder L can be expressed as in equation (1).

$$\frac{x - a}{l} = \frac{y - b}{m} = z \qquad (1)$$

Where l, m, and 1 are the components of the directional vector $\vec{L}$ (l, m, 1) in the x, y, and z directions.

The distance between any measurement point
$$p_i = (x_i, y_i, z_i), \ i = (1, 2, \ldots, N)$$ and the axis is given by
equation (2)

$$R_i = \sqrt{[x_i - (lz_i + a)]^2 - [y_i - (mz_i + b)]^2} \qquad (2)$$

When MZC evaluation technique is used to evaluate the cylindricity error; mathematically, the estimation of the cylindricity value can be formulated as an optimization problem; $\min[\max(R) - \min(R)]$. Hence the fitness function for a MZC is derived below, equation (3).

$$f(a, b, l, m) = \min[\max(R_i) - \min(R_i)] \qquad (3)$$

Using the results of the PSO optimization problem; (a, b, l, m), the cylindricity error can be written as Eq. (4):

$$\delta = \sqrt{[x_1 - (lz_1 + a)]^2 - [y_1 - (mz_1 + b)]^2} - \sqrt{[x_2 - (lz_2 + a)]^2 - [y_2 - (mz_2 + b)]^2} \qquad (4)$$

Where the farthest and the nearest points from the cylinder axis respectively are: (x_1, y_1, z_1) and (x_2, y_2, z_2).

IV. PARTICLE SWARM OPTIMIZATION TECHNIQUE PSO

Particle swarm optimization technique was originally developed to simulate the search procedure of a bird flock or fish school to find their food sources. Now, PSO is basically a method for optimization of continuous nonlinear functions. In this technique, the system is initialized with a population of random solutions; each solution is called a particle. Particles are moving and hence have a velocity. Each particle remembers the position it was in having the best result so far (its personal best). But, this would not be much good on its own; particles need to cooperate together in figuring out where to search [14]. In every iteration each particle updates its velocity vector and position based on two ''best'' values. The first one is the best solution that it has achieved so far. This value is a personal best called ''pbest''. The other ''best'' value is the best value obtained so far by any particle. This best value is a global best and is called ''gbest'' [14]. Suppose the search space is d-dimensional, the j-th particle of the population can be represented by a d-dimensional vector $x_j = (x_{j1}, x_{j2}, x_{j3}, \ldots x_{jd})$. The velocity of this particle can be represented by another d-dimensional vector $v_j = (v_{j1}, v_{j2}, v_{j3}, \ldots v_{jd})$. The best position of the j-th particle visited previously is denoted by $p_j = (p_{j1}, p_{j2}, p_{j3}, \ldots p_{jd})$. The best position of all particles is denoted by $p_g = (p_{g1}, p_{g2}, p_{g3}, \ldots p_{gd})$. Each particle updates its velocity and position according to the following two equations, eqs. (5):

$$\begin{cases} V_{id}^{k+1} = w \times v_{id} + c_1 \times ran_1 \times (P_{id} - x_{id}) + c_2 \times ran_2 \times (P_{gd} - x_{id}^k) \\ x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \end{cases} \qquad (5)$$

Where w is the inertia weight; c_1 and c_2 are learning factors; ran_1 and ran_2 are random numbers distributed uniformly in the range [0, 1]. For the different optimization issues, the parameters of PSO can be adjusted to improve the convergence of the algorithm.

A computer program has been developed using MATLAB to evaluate cylindricity error of the tested cylinders using the data collected by the CMM. The developed program was verified by evaluating cylindricity errors for different measurement runs of the same cylinder. Results were also compared with corresponding values estimated by the CMM software. Results show very good repeatability of the developed Matlab program. A flow chart of the developed program is shown in Figure 4.
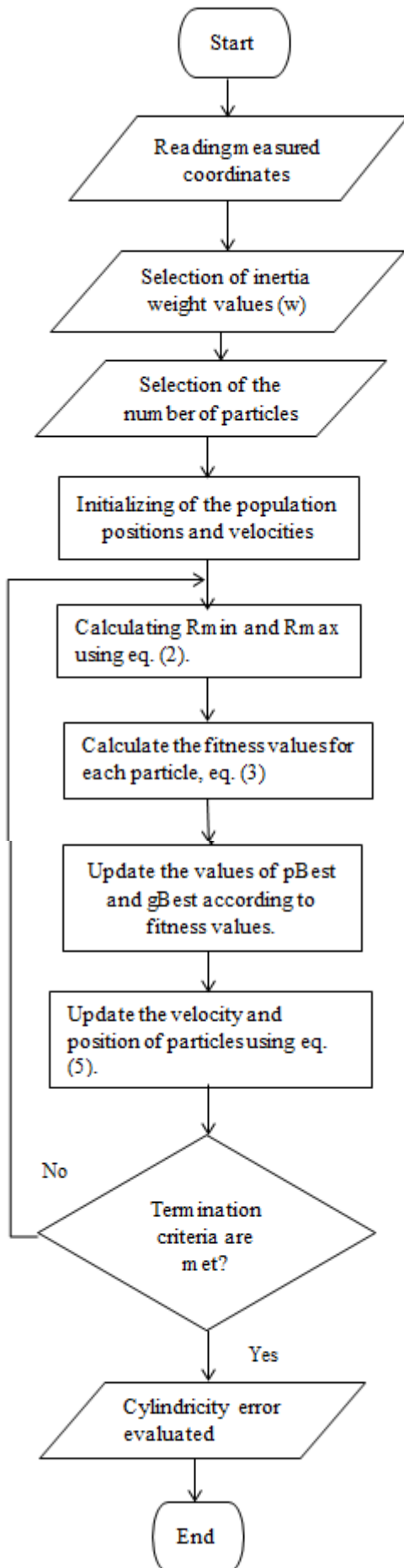
Figure 4 Developed computer program steps

The sampling rate used during measurement was 20 readings/ mm which gives a step width of 0.05mm. Changing the sampling rate will lead to a new sample density and can affect the value of the estimated cylindricity error. Figure 4 shows screenshots for the probed circles and helix respectively. Table 1 shows the sampling step width and the corresponding cylindricity error when estimated using circles strategy with ten circles on each measurement. Table 2 shows the results when runs of 10 helix each were performed on the same cylinder. Figures 5 (a, b) show the estimated cylindricity error at different sampling step widths for both circles and helix strategies respectively.
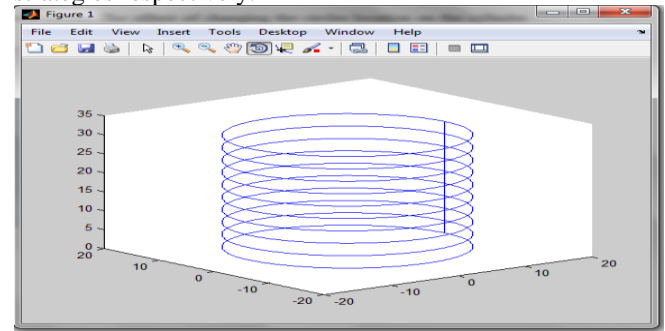
(a)
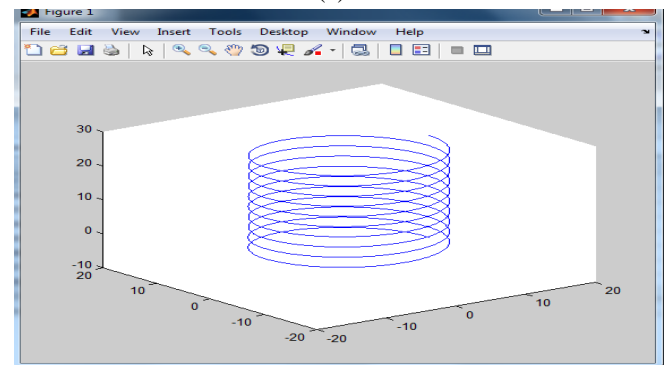
(b)

Figure 5 Screenshots for the probed circles (a) and helix (b).

Table 1 Cylindricity errors using circles strategy at different sampling rates

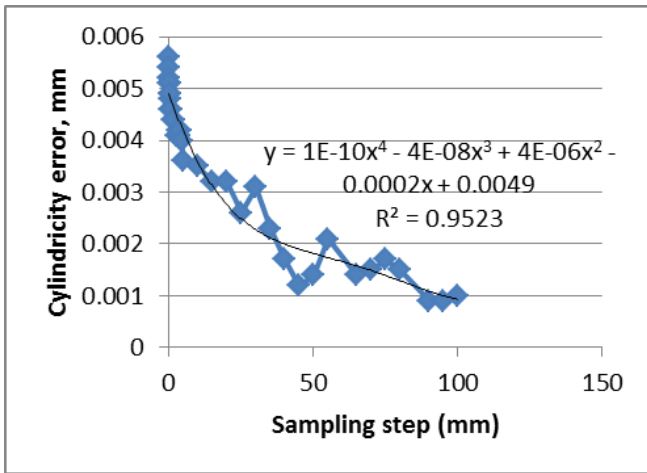| step width (mm)	Avg. error (mm)	step width (mm)	Avg. error (mm)
0.05	0.0056	10	0.0035
0.1	0.0054	15	0.0032
0.15	0.0051	20	0.0032
0.2	0.0052	25	0.0026
0.25	0.0051	30	0.0031
0.3	0.0051	35	0.0023
0.35	0.0048	40	0.0017
0.4	0.0049	45	0.0012
0.45	0.0049	50	0.0014
0.5	0.0046	55	0.0021
1	0.0046	65	0.0014
1.5	0.0044	70	0.0015
2	0.0044	75	0.0017
2.5	0.0041	80	0.0015
3	0.0042	90	0.0009
3.5	0.0041	95	0.0009
4	0.0042	100	0.001
4.5	0.004	95	0.0009
5	0.0036	100	0.001

Figure 6 Effect of sample rate variation on evaluated
cylindricity errors (circles strategy)

Table 2 Cylindricity errors using helical strategy
at different sampling rates (10 helix turns)

| step width (mm)	Avg. error (mm)	step width (mm)	Avg. error (mm)
0.05	0.0053	10	0.0035
0.1	0.0051	15	0.0029
0.15	0.0047	20	0.003
0.2	0.0049	25	0.0028
0.25	0.005	30	0.0025
0.3	0.0045	35	0.0025
0.35	0.0047	40	0.0022
0.4	0.0048	45	0.0027
0.45	0.0047	50	0.0022
0.5	0.0046	55	0.0018
1	0.0043	60	0.0021
1.5	0.0042	65	0.0022
2	0.004	70	0.002
2.5	0.0041	75	0.0015
3	0.0039	80	0.0015
3.5	0.0039	85	0.0016
4	0.004	95	0.0009
4.5	0.0038	100	0.0011
5	0.0038	100	0.0011
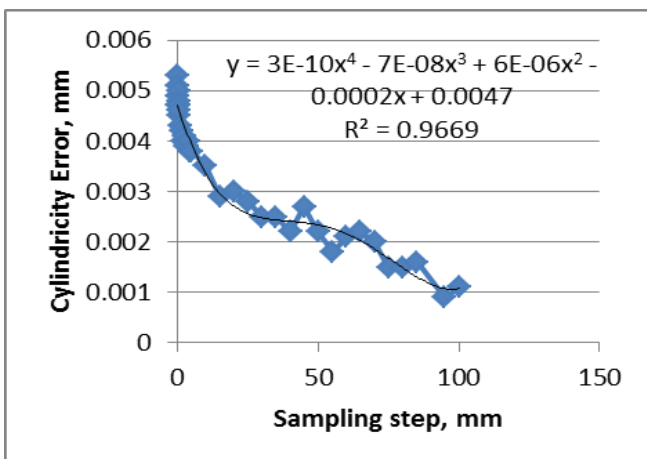
Figure 7 Effect of sample rate variation on evaluated
cylindricity error (Helical strategy).

The obtained results as shown in Table (1) and Figure (6) reveal that the optimized cylindricity error gives highest estimated value at the minimum sample width of 0.05 mm when using circles strategy. Similar results are also observed when helical strategy is used, Table (2) and Figure (7). In both cases, increasing the sampling step width will lead to a relative decrease in the value of the estimated cylindricity error. It is notable that increasing the sampling step width from 0.05 mm to 0.5 mm leads to a 17.85 % reduction in the estimated cylindricity error for circles strategy and a 13.2% reduction for helical strategy. It is also clear that increasing the step width from 0.5 mm to 5 mm caused another reduction in the cylindricity error value by 21.70% for circles strategy and 17.39% reduction for helical strategy. It could be concluded that using smaller step width insures a more complete coverage for the cylinder surface with all geometrical deviations resulting in a more accurate error evaluation.

Comparison of results shown in tables (1) and (2), reveals slight variations in the estimated error using circles and helical strategies, for the same test cylinder, when 10 circles and 10 helix turns are used. Close results are obtained when using an equal number of circles and helix turns, e.g. at step width of 0.05 mm the difference is 0.3 µm. This indicates that for the considered parts both methods can be used alternatively, but the circles strategy is slightly time consuming than the helical one.

To investigate the effect of the number of probed circles on the estimated value of cylindricity error, a test cylinder was scanned around 20 circular sections to get a high density surface coordinate data file. The data file was then divided into subsets of surface coordinate data files with different number of circles. The subset files were processed with the developed Matlab program to evaluate cylindricity error and results obtained from subset files were compared. Table (3) shows cylindricity errors evaluated using different subsets of data files.

Table 3 Cylindricity error for the same cylinder
evaluated using different number of circles and the same
sampling rate

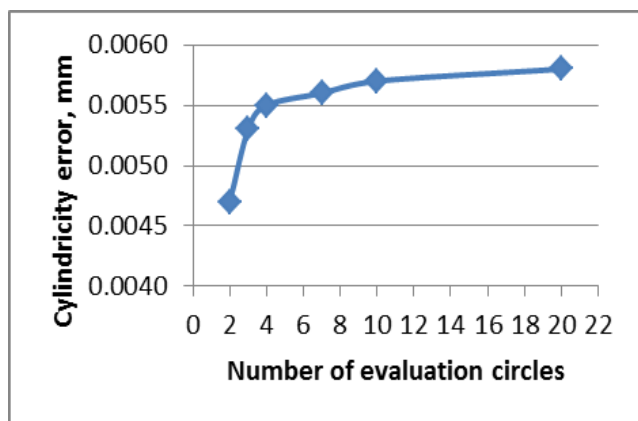| No. of evaluation circles	Included circles	Cylindricity error (mm)	Percentage of increase (to 2 circles)
20	All	0.0058	23
10	1, 3, 5, 7, 9, 11, 13, 15, 17, 19	0.0057	21
7	1, 4, 7, 10, 13, 16, 19	0.0056	19
4	1, 7,13, 19	0.0055	17
3	1, 10, 19	0.0053	13
2	1, 11	0.0047	0

Figure 8 Effect of number of measurement circles on evaluated cylindricity error for the same cylinder.

Figure 8 shows the effect of the number of measurement circles on the evaluated cylindricity error for the same cylinder keeping the sampling rate at 20 samples/mm. The cylindricity error obtained when considering 20 circles is 0.0058 mm. As the number of circles decreases the estimated cylindricity error decreases, however when considering 5 to 7 circles the error remains almost constant (0.0056 mm). Although the total number of evaluation data points is decreased, estimated cylindricity error shows a very small change of 0.0002 mm which is relatively very small. When two circles were used in the evaluation, a significant reduction is observed in the value of the estimated error. Thus it can be concluded that, to get accurate evaluation of cylindricity error a minimum of 5 circular sections should be probed and considered provided that the sampling rate is adequate.

VI. CONCLUSION

This study investigates the effect of measurement strategy and evaluation data density on the value of estimated cylindricity error. Three cylinders were scanned using two different strategies, circles and helical. Data density was changed by changing either the sample width (sampling rate) or the number of probed circles. Sampling width was varied between 0.05 and 100mm, and the number of probed circles was also changed between 2 and 20 circles/turns. For the same sampling rate, estimates of cylindricity error were very close when using either circles or helix strategies. Both circles and helix strategies can be used alternatively however, helix strategy is slightly faster.

Changing data density either by changing sampling rate or number of circles/helix turns changes the value of estimated cylindericity error. The change in sampling rate has more pronounced effect on estimated error which is quite similar for both strategies. It was observed that, for the considered part, a change in the sampling step width from 0.05 mm to 0.5 mm caused a reduction of 17.85% in the estimated value of error when using circles strategy. Meanwhile, for the helical strategy the corresponding reduction was 13.2%. Further reductions of 21.7% and 17.39% were reported for circles and helix strategies respectively when the sampling step width

was increased from 0.5mm to 5mm.This means that using smaller sampling step width gives more accurate estimate of cylindricity error.

On the other hand increasing the number of probed sections gives higher values for the estimates of cylindericity error. At the same sampling step width, moving from 2 to 4 evaluation sections results in an increase of approximately 17% in the estimated error. If the number of evaluation sections becomes 7, this relative increase becomes 19% (approx.). At 20 evaluation sections the relative increase is 23% (approx.). Starting from 4 circular sections, the rate of increase in the value of the estimated error becomes slower. Compared to the estimated error at 4 sections, the relative increase at 20 sections is only 5.4 % higher (approx.). Although there are slight changes between specimens in the percentage increase of the estimated error, but they all have the same trend. Based on the obtained results, 5-7 sections will provide quite accurate evaluation of cylindericity error provided that the sampling rate is adequate.

The results show that sampling rate variation has much greater impact on the value of estimated cylindricity error compared with the number of measurement sections for both circles and helical strategies irrespective of the actual number of evaluation points.

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